B.Sc Part II, Physics (Hons)

Dr Satyadeo Narayan Singh

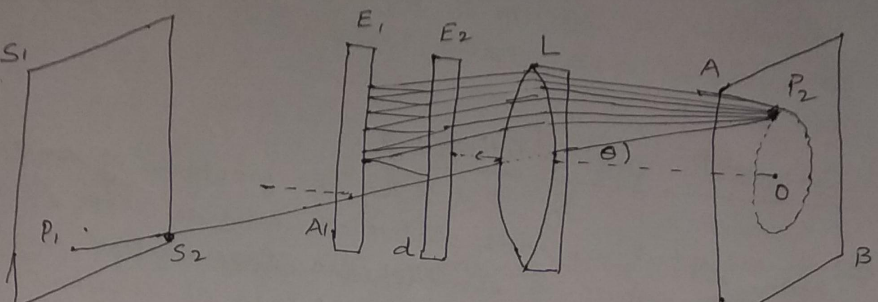
S.B. College, Ara

①

Q Give the theory of Fabry-Perot Interferometer and obtain an expression for its intensity distribution in fringes.

Ans Febry-Perot ~~Inf~~ Interferometer :→

It is high R.P instrument which utilises the fringes produced in the transmitted light after multiple reflection in the air film between two plane plates thinly silvered in the inner surfaces.



Construction :-

E_1, E_2 → Two optically plane glass plates with their inner surfaces silvered, and placed accurately parallel to each other. The plates are made slightly prismatic to avoid interference among the rays reflected at the outer unsilvered surfaces. One plate is fixed while the other may be moved toward or away from it on a carriage riding on accurately machined ways by a slow motion screw.

S_1S_2 :→ A broad source of monochromatic light, giving parallel incident rays on E_1

L ⟶ A convex lens to bring the transmitted parallel rays togather at a point P_2 in its focal plane where they interfere.

Action :→ An incident ray P_1A_1 suffers multiple-reflection at the two silvered surface of E_1 and E_2. At each reflection, a part of light is transmitted also. Each incident ray is producing a group of coherent parallel transmitted rays with a constant path difference between two successive rays which interfere in focal plane of L.

If t = ~~thickness~~ seperation between the plates
θ = angle of incident then the path difference between two successive transmitted rays is $2t\cos\theta$.

For constructive Interference :→

$$2t\cos\theta = n\lambda$$

Here λ = wavelength of light and n = order of fringes. The focus of point in the source which give ray of a constant θ is a circle. Hence the interference pattern consists of a system of bright concentric rings on a dark background each ring corresponds to a ~~partict~~ particular value of θ.

Intensity Distribution :→

If T & R be the fractions of intensity transmitted and reflected at each silvered surface then $\sqrt{T}$ and $\sqrt{R}$ will be the fractions of amplitudes transmitted and reflected respectively. If 'a' is the amplitude of incident wave then the amplitude of successive transmitted ray through E_1, E_2 will be aT, aTR, aTR^2 - - - - - - -

The phase difference between two successive rays is

$$\delta = \frac{2\pi}{\lambda}(2t\cos\theta)$$

Denoting the incident wave by $y = ae^{i\omega t}$, the transmitted wave is

$$y_1 = aTe^{i\omega t} = \cancel{aT}, \quad y_2 = aTRe^{i(\omega t-\delta)}$$

$$y_3 = aTR^2e^{i(\omega t-2\delta)} \text{ and so on - - - - - -}$$

The resultant amplitude will be

$$A = aT + aTRe^{-i\delta} + ATR^2 \cdot e^{2i\delta} + aTR^3 \cdot e^{3i\delta} + \cdots\cdots$$

$$= aT(1 + Re^{i\delta} + R^2e^{2i\delta} + R^3 \cdot e^{3i\delta} + \cdots)$$

$$= aT\frac{1}{1-Re^{-i\delta}}$$

The complex conjugate of A is

$$A^* = a \cdot T \cdot \frac{1}{\cancel{B}1 - Re^{-i\delta}}$$

The resultant intensity is given by

$$I = A \cdot A^* = \frac{a^2T^2}{(1-Re^{-i\delta})(1-Re^{+i\delta})}$$

$$= \frac{a^2T^2}{1+R^2-2(e^{-2\delta}+e^{2\delta})} = \frac{a^2T^2}{1+R^2-2R\cos\delta}$$

$$= \frac{a^2T^2}{(1-R)^2+2R(1-\cos\delta)} = \frac{a^2T^2}{(1-R)^2+4R\sin^2\frac{\delta}{2}}$$

$$= \frac{a^2T^2}{(1-R)^2}\left[\frac{1}{1+\frac{4R}{(1-R)^2}\cdot\sin^2\frac{\delta}{2}}\right]$$

For Maximum Intensity :→

$\sin^2\frac{\delta}{2} = 0$. or $\frac{\delta}{2} = n\pi$ or $\delta = 2n\pi$

$$\therefore I_{max} = \frac{a^2T^2}{(1-R)^2}$$

For Minimum Intensity :→

$\sin^2\frac{\delta}{2} = 1$ or $\frac{\delta}{2} = (2n+1)\cdot\pi/2$

$\therefore \delta = (2n+1)\pi$

$$I_{min} = \frac{a^2T^2}{(1-R)^2}\cdot\frac{1}{1+\frac{4R}{(1-R)^2}} = \frac{a^2T^2}{(1+R)^2}$$

Hence, the intensity expression for F.P fringes is

$$I = \frac{I_{max}}{1+\frac{4R}{(1-R)^2}\cdot\sin^2\frac{\delta}{2}}$$

The plot of I against δ for different values of R is shown. This shows that larger the value of R, the more rapid is the fall of intensity on either side of its maximum and greater is the difference between I_{max} and I_{min}. Hence we obtain a system of sharp and bright ~~fringe~~ rings against a wide dark background.